

# Estimation of Turc reference evapotranspiration with limited data against the Penman-Monteith Formula in Senegal

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*Submitted on 2015, 17 December; accepted on 2016, 17 May. Section: Research Paper*

**Abstract:** Estimation of daily evapotranspiration ( $ET_0$ ) is required for efficient irrigation management in Senegal but physically based equations such as the Penman–Monteith (PM) approach need large inputs data which are not always available. Therefore empirical approaches such as Turc’s formula are often used. Unfortunately, This latter showed high annual  $ET_0$  compared to FAO Penman–Monteith (FAO-PM)  $ET_0$  for stations in this region. Moreover, Turc underestimates mean daily  $ET_0$  for the dry season and overestimates  $ET_0$  for the wet season. For reevaluation, Turc’s empirical parameters ( $a$  and  $b$ ) were fitted to match FAO-PM  $ET_0$ , whereby the fitting did not improve the prediction substantially and only assuming  $C$  being independent on relative humidity yielded better agreement. In a second step universal  $a$  and  $b$  parameters were estimated for different stations in Senegal. The results indicate that one set of parameters ( $a$  and  $b$ ) can be used to predict all station  $ET_0$  with fairly high accuracy indicated by a  $R^2$  of 0.80 and an RMSE of 0.71 mm day<sup>-1</sup>. Based on these findings we propose to use Turc’s approach in Senegal only with the parameters  $a = 0.16$  and  $b = 29.19$  with the constrain that  $C$  is independent on relative humidity.

*Keywords:* reference evapotranspiration, Turc formula, Penmen-Monteith, Senegal

## Introduction

Water scarcity is increasingly becoming the most important environmental constraint limiting plant growth in many semi-arid regions. Therefore knowledge of

evapotranspiration, which involves the evaporation of water from land surfaces and transpiration by vegetation, is essential for estimating optimal irrigation water practices requirements (Allen *et al.*, 1998). In most cases, reference evapotranspiration ( $ET_0$ ) has been computed by the Penman-Monteith equation (PM) as recommended by the FAO and applied over different climates over the globe (Penman, 1948; Allen *et al.*, 1998, 2006; Garcia *et al.*, 2004). Unfortunately, estimation of reference  $ET_0$  by the physically based Penman-Monteith (PM) equation is largely limited by the availability of the input meteorological data needed for the calculation, even if the FAO-56 (Allen *et al.*, 1998) provided easy lookup tables or approximations if some measurements are not available. On the other hand, there are also simplified PM or empirical approaches documented to estimate  $ET_0$  such as the approach of Priestly-Taylor (Priestly and Taylor, 1972) and Makkink (Makkink, 1957), which are both a simplification of the Penman-Monteith equation, or the empirical models of Hargreaves (Hargreaves and Samani, 1985), Thornthwaite (Thornthwaite, 1948), or Turc (Turc, 1961). In general, the Penman-Monteith equation as described in FAO-56 (FAO-PM) presents two main advantages over the others: 1) it is physically-based, and can therefore be globally applied without any adjustment of input parameters, 2) it is well documented, implemented in a wide range of software, and has been calibrated by means of lysimeters (Droogers and Allen, 2002). That is why it is frequently cited as the preferred method for the calculation of  $ET_0$ , especially for calculations at short temporal scales (Alexandris and Kerkides, 2003). Thus, the fair results obtained in many different studies at daily to longer temporal scales is surprising even if the combined equation was theoretically derived for instantaneous values of the variables involved (Allen *et al.*, 2006). However, it requires several measurements of climatic variables such as air temperature, wind speed, relative humidity, soil heat flux, and solar radiation which are not measured at hourly to daily basis in many stations especially in developing countries (Irmak *et al.*, 2003; Gavilán *et al.*, 2006). Despite the attempts of Allen *et al.* (1998) to estimate solar radiation and humidity from other variables easier to measure, it is difficult to obtain the required accuracy without modern electronic devices, especially those providing wind speed and air vapor pressure values. Moreover, the lack of reliable measurements in areas where  $ET_0$  estimates are especially needed is very common (Allen and Pruitt, 1986; Liu and Todini, 2002; Maeda *et al.*, 2010). These shortcomings in the application of FAO-PM equation motivated the derivation of less demanding models in terms of input data such as the Turc equation, where only limited data are required. Several authors have reported that the Turc equation, which was originally developed for Mediterranean countries, tends to overestimate  $ET_0$  for humid locations (Mohammad, 1978; Jensen *et al.*, 1990). However, the Turc method was considered by many authors (Schoch, 1965; Cornet, 1977; Tandia, 1989; Dacosta, 1989; Gaye, 1990) as the best model to estimate  $ET_0$  in Senegal. This model yielded also the best

estimate of the reference evapotranspiration among five others empirical methods for three stations located in eastern North Carolina, USA (Amatya *et al.*, 1995). Because application of the original Turc formula for several climatic stations in Senegal showed that the calculated  $ET_0$  is high compared to reference  $ET_0$  based on the FAO-PM equation and that the mismatch between Turc and FAO-PM is seasonal depended, leads to the assumption that the original parameters and constraints used in the Turc formula are not valid for Senegal. This motivated our research to evaluate the Turc formula in detail for the application for Senegal. Therefore, we first analyzed which parameters and constraints in the Turc formula have to be adapted and if the fitted new parameters and assumptions are valid for different stations across Senegal.

## Materials and methods

### *Area description and weather data*

The climate in Senegal is humid in the southern part and semiarid in the North with mean annual rainfall ranging between 320 to 1200 mm per year. In general, 80 % of the annual rainfall occurs during the months August to September. On the other hand, differences in the mean annual temperature are not very large.

For our study we selected five weather stations at different locations in Senegal (Fig.1). These stations were chosen because data were available for a sufficient long

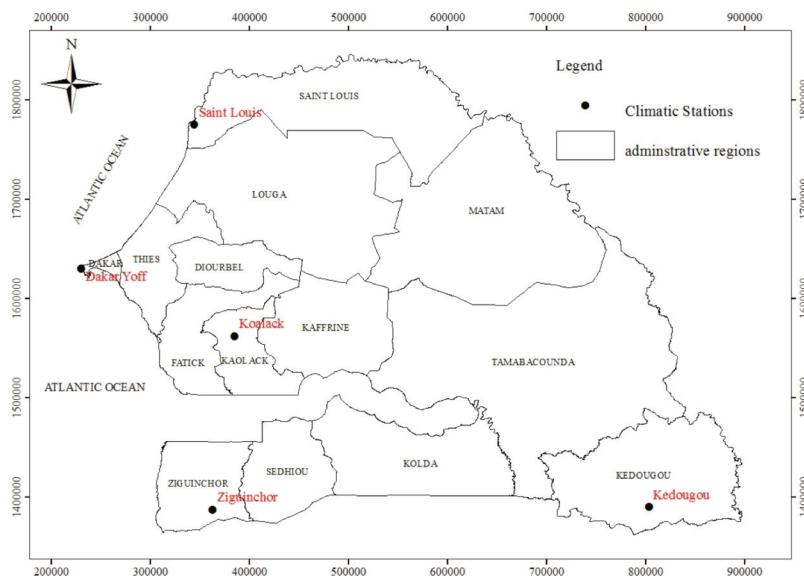


Figure 1 - Localization of the weather stations in Senegal used in this study.

*Table 1 - Basis climatic data for the time period 2000 to 2013 for the five selected stations shown in Fig. 1.*

STATIONS	LONGITUDE	LATITUDE	ALTITUDE [m]	TEMPERATURE [°C]		RAINFALL [mm/year]
				max	min	
Dakar-Yoff	17.50 W	14.73 N	27	28.3	22.5	489.8
Kaolack	16.07 W	14:13 N	6	36.8	23.1	678.8
Saint Louis	16.45 W	16:05 N	4	32.5	21.3	328.7
Kedougou	12:22W	12:57 N	178	35.7	22.3	951.7
Ziguinchor	16:27 W	12:55 N	26	35.1	21.8	1168.7

time period (years 2000 to 2013) and a full set of meteorological data necessary for the calculation of reference Penman-Monteith  $ET_0$  were available without any gaps. Additionally, these stations cover a broad spectrum of climatic conditions of Senegal from Atlantic influenced (maritime) to more continental. Basic climatic data of these stations are listed in Tab. 1.

As can be seen rainfall in the years 2000 to 2013 ranged from 329 mm for the Saint Louis station in the North to 1169 mm recorded at the Ziguinchor station in the South-West. Annual maximum and minimum temperatures for these five locations vary between 28.3 to 36.8 °C and 21.3 to 23.1 °C, respectively. Generally, maximum temperatures occur during the months July, August, September, and October corresponding to the rainy season. Mean daily FAO-PM evapotranspiration varies strongly between 2 to 4; 1 to 5; 2 to 5; and 2 to 7 mm for Dakar-Yoff, Ziguinchor, Saint Louis, Kedougou, and Kaolack stations, respectively.

In this study, full climatic data sets were collected from the Senegal National Civil Aviation and Meteorological Agency (ANACIM). Weather data included daily values of maximum and minimum air temperature, relative humidity, wind speed, and sunshine hours. Unfortunately, radiation data was not provided for all stations.

### **Reference Evapotranspiration ( $ET_0$ )**

#### *The FAO-56 Penman-Monteith equation*

According to Allen *et al.* (1998), the estimation of  $ET_0$  using Penman-Monteith approach can be written as:

$$ET_0 = \frac{0.408\Delta(R_n - G) + \gamma \frac{900}{T + 273} u_2 (e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \quad (1)$$

where  $ET_0$  is the reference evapotranspiration ( $\text{mm day}^{-1}$ ),  $R_n$  is the daily net radiation ( $\text{MJ m}^{-2} \text{day}^{-1}$ ),  $G$  is the soil heat flux ( $\text{MJ m}^{-2} \text{day}^{-1}$ ),  $T$  is the average daily air

temperature at a height of 2 m ( $^{\circ}\text{C}$ ),  $u_2$  is the daily mean wind speed at a height of 2 m ( $\text{m s}^{-1}$ ),  $e_s$  is the saturation vapor pressure (kPa),  $e_a$  is the actual vapor pressure (kPa),  $\Delta$  is the slope of the saturation vapor pressure versus the air temperature curve ( $\text{kPa } ^{\circ}\text{C}^{-1}$ ), and  $\gamma$  is the psychrometric constant ( $\text{kPa } ^{\circ}\text{C}^{-1}$ ).

Because  $R_n$  was not provided by all stations,  $R_n$  was approximated using Eq. 40 in FAO-56 by:

$$R_n = R_{ns} - R_{nl} \quad (2)$$

where  $R_{ns}$  is the incoming net shortwave radiation ( $\text{MJ m}^{-2} \text{ day}^{-1}$ ) and  $R_{nl}$  the outgoing net longwave radiation.

The incoming net shortwave radiation can be calculated using Eq. 38 in FAO-56:

$$R_{ns} = (1 - \alpha) * R_s \quad (3)$$

where  $\alpha$  is the albedo or canopy reflection coefficient set to 0.23, and  $R_s$  is the incoming solar radiation ( $\text{MJ m}^{-2} \text{ day}^{-1}$ ).

Secondly,  $R_s$  had to be calculated by Eq 35 in FAO-56:

$$R_s = \left( a_s + b_s * \frac{n}{N} \right) * R_a \quad (4)$$

where  $a_s$  was set to 0.25 and  $b_s$  to 0.5.  $n$  is the actual duration of sunshine (h),  $N$  is the maximum possible duration of sunshine (h), and  $R_a$  is the extraterrestrial radiation ( $\text{MJ m}^{-2} \text{ day}^{-1}$ ).

Finally, the net outgoing longwave radiation was calculated based on Eq. 39 in FAO-56:

$$R_{nl} = \delta \left( \frac{T_{max,k^4} + T_{min,k^4}}{2} \right) (0.34 - 0.14\sqrt{e_a}) \left( 1.35 \frac{R_s}{R_{so}} - 0.35 \right) \quad (5)$$

where  $R_{nl}$  is the net outgoing longwave radiation [ $\text{MJ m}^{-2} \text{ day}^{-1}$ ],  $\sigma$  is the Stefan-Boltzmann constant [ $4.903 \cdot 10^{-9} \text{ MJ K}^{-4} \text{ m}^{-2} \text{ day}^{-1}$ ],  $T_{max}$  and  $T_{min}$  are the maximum and minimum absolute daily temperatures [K],  $e_a$  is the actual vapor pressure [kPa],  $R_s/R_{so}$  is the relative shortwave radiation (limited to  $\leq 1.0$ ),  $R_s$  is the solar radiation [ $\text{MJ m}^{-2} \text{ day}^{-1}$ ] [Eq. 4], and  $R_{so}$  is the clear-sky radiation [ $\text{MJ m}^{-2} \text{ day}^{-1}$ ].

It has to be noted that the soil heat flux ( $G$ ) was assumed to be zero over the calculation time step period of 24 hours as recommended by Allen *et al.* (1998).

### **Turc formula**

The Turc formula (Turc, 1961) was originally developed for southern France and northern Africa. It is based on some easily available climatic data such as radiation, air temperature, and relative humidity, and therefore, easy to apply whenever a full set of climatic data is not available. The Turc equation for daily potential evapotranspiration calculation is given by Eq. [6]:

$$ET_0 = a * C * (R_G + b) \frac{T}{T+15} \quad (6)$$

where  $ET_0$  is in  $\text{mm day}^{-1}$ ,  $T$  is the mean daily air temperature ( $^{\circ}\text{C}$ ),  $R_G$  is the global radiation ( $\text{MJ m}^{-2} \text{day}^{-1}$ ),  $a$  and  $b$  are empirical constants with  $a = 0.31 (\text{m}^2 \text{MJ}^{-1} \text{mm}^{-1})$  and  $b = 2.094 (\text{MJ m}^{-2} \text{day}^{-1})$ .

Additionally, the parameter  $C$  is constrained by the relative humidity  $RH$  [%] by:

$$C = 1 + \frac{50 - RH}{70} \quad \text{if } RH < 50 \% \quad (7)$$

$$C = 1 \quad \text{if } RH \geq 50 \% \quad (8)$$

Physically, this constrain means that the overall  $ET_0$  increases linearly at given temperature and radiation with decreasing relative humidity below the threshold of 50%  $RH$ . Therefore, the term  $C$  can be somehow related to the vapor deficit term as described in the PM equation [Eq. 1]. On the other hand, at higher relative humidity  $ET_0$  is mainly driven by radiation and temperature and relative humidity does not play a role anymore.

Again an approximation for the global radiation is provided if not measured directly by knowledge of the sunshine duration Eq. [9]:

$$R_G = R_o * \left( 0.19 + 0.55 * \frac{S}{S_0} \right) \quad (9)$$

where  $R_o$  is the extraterrestrial radiation ( $\text{MJ m}^{-2}$ ),  $S$  sunshine duration (h), and  $S_0$  is the astronomic possible sunshine duration (h).

## Statistical analysis

Quantitative approaches to evaluate the model performance were applied. To ensure a rigorous comparison of the methods, an extended analysis was performed using different statistical indices for the estimated values. The  $R^2$  as the square of the Pearson's correlation coefficient (Eq. 10) was used as well as the adjusted  $R^2$  ( $R^2_{adj}$ ) accounting for different degrees of freedom or here number of fitting parameters.

$$R^2 = \frac{[\sum_{i=1}^n (x_i - \bar{x}) * (y_i - \bar{y})]^2}{\sum_{i=1}^n (x_i - \bar{x})^2 * \sum_{i=1}^n (y_i - \bar{y})^2} \quad (10)$$

$$R^2_{adj} = 1 - \frac{(1 - R^2)(N - 1)}{N - p - 1} \quad (11)$$

where  $R^2$  will be calculated by Eq. [10],  $p$  the number of fitted parameters, and  $N$  is the total number of observations.

Finally the root mean squared error (RMSE) (Eq. 12) was calculated. In general, the RMSE can range from 0 to infinity, and of course, lower values indicate better agreement between the two data sources (Willmott, 1981).

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (x_i - y_i)^2}{N}} \quad (12)$$

where  $x_i$  stands for estimated values by FAO-PM method and  $y_i$  stands for values predicted by the compared Turc method.

## Results

As mentioned above, the evaluation of Turc method (Eq. 6) was performed by comparison with the FAO-PM equation. This evaluation followed two steps. In the first one,  $ET_0$  from the Turc empirical method was computed with the original parameter values given above. In the second stage, Turc's empirical parameters  $a$  and  $b$  (as well as the new introduced parameters  $e$  and  $f$ ) were fitted based on reference  $ET_0$  from the FAO-PM method.

### Using Turc original formula to predict $ET_0$

In a first step, daily  $ET_0$  ( $\text{mm day}^{-1}$ ) for five different synoptic weather stations located in Senegal were computed for the period 2000 to 2013 using Turc formula and the FAO-PM method. The  $ET_0$  values estimated by Turc empirical equations were then compared with the estimates provided by the physically based standard FAO-PM equation. FAO-PM was selected as a standard method for comparison because it is a globally accepted model, used under a variety of climatic regimes and reference conditions.

The mean daily  $ET_0$  values for each month averaged over the 14 years calculated by both methods for all five locations are plotted in Fig. 2 and the statistics are provided in Tab. 2. As can be seen in Fig. 2  $ET_0$  calculated by the original Turc formulation generally overestimates FAO-PM at all stations. Additionally, the Turc method under predicts mean daily  $ET_0$  for the dry season, especially for the stations located in Saint-Louis, Kaolack, and Kedougou, whereas an overestimation is mainly detectable within the wetter season. This behavior is in good agreement with observations reported by Dacosta (1989) at Kolda station. In the same order of magnitude, Djaman *et al.* (2015) concluded that Turc formula generally underestimates  $ET_0$  in a study conducted in two continental stations (Ndiaye and Fanaye) located in the Senegal River Valley using approximately 1 years time period.

Table 2 - Summary statistics of daily  $ET_0$  estimated by Turc method against that estimated by the FAO-PM method.

	ORIGINAL TURC FORMULATION (a=0.31, b=2.094)		TURC FORMULATION WITH FITTED a AND b PARAMETERS					TURC FORMULATION WITH FITTED a, b, e, and f PARAMETERS (Eq.13)						TURC FORMULATION WITH FITTED A AND b PARAMETERS WITHOUT CONSTRAINING C (Eq.13) (e=50 and f=70)					
	R <sup>2</sup>	RMSE	a	b	R <sup>2</sup>	R <sup>2</sup> <sub>adj</sub>	RMSE	a	b	e	f	R <sup>2</sup>	R <sup>2</sup> <sub>adj</sub>	RMSE	a	b	R <sup>2</sup>	R <sup>2</sup> <sub>adj</sub>	RMSE
Dakar-Yoff	0.07	1.58	0.07	52.73	0.16	0.16	0.95	0.15	36.65	48.76	68.30	0.90	0.895	0.33	0.14	38.39	0.90	0.89	0.33
Saint Louis	0.22	1.37	0.11	36.86	0.44	0.43	1.14	0.17	28.07	50.37	70.00	0.86	0.86	0.52	0.17	28.00	0.86	0.86	0.52
Ziguinchor	0.51	1.34	0.20	5.48	0.53	0.53	0.81	0.15	21.04	51.60	79.12	0.82	0.81	0.50	0.14	25.45	0.81	0.81	0.52
Kaolack	0.40	1.53	0.21	12.27	0.47	0.47	1.49	0.18	23.00	55.24	63.02	0.77	0.77	0.94	0.20	22.06	0.77	0.77	0.94
Kedougou	0.53	1.14	0.18	15.36	0.63	0.63	1.05	0.12	43.21	47.84	77.40	0.86	0.86	0.60	0.11	47.14	0.86	0.86	0.60



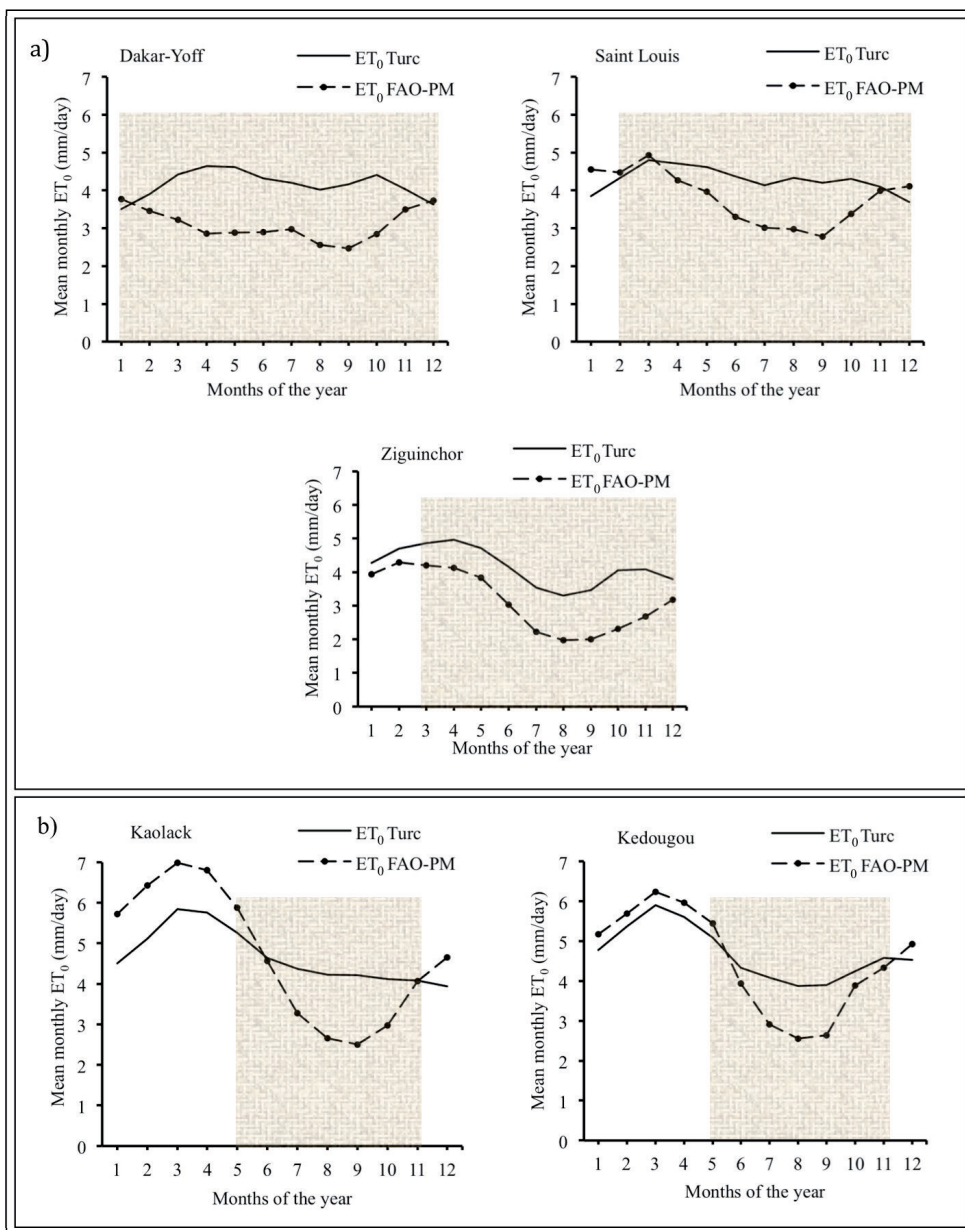


Figure 2 - Comparison of monthly mean daily  $ET_0$  computed by Turc method against FAO Penman-Monteith (FAO-PM) method, a: coastal stations and b: continental stations. Note that the monthly values are the mean of 13 consecutive years (2000- 2013) (the gray area represent the months where the humidity is  $\geq 50\%$ ).



The reason for the mismatch in our study can be found in the higher wind speed which exceeded  $3 \text{ m s}^{-1}$  during the dry period, which lead to high values for the aerodynamic term (advection) in the FAO-PM, whereby the Turc method does not account for wind speed at all (Berengena and Gavilán, 2005). Additionally, the large mismatch between the two approaches during the wet season can be explained by the fact that the constrain of the Turc approach (Eq. 7 and 8) will not scale the  $ET_0$  by  $C$  at relative humidity exceeding a certain threshold (here 50% RH).

However, this seasonally trend is less pronounced for the costal stations (Dakar-Yoff, Ziguinchor, and Saint Louis), where the relative humidity is generally larger than 50 % over the entire year.

Coming back to the statistics listed in Tab. 2 the stations ranked first with the lowest root mean square error (RMSE) and the highest coefficient of determination ( $R^2$ ) for daily  $ET_0$  prediction using Turc equation against FAO-PM are the stations at Kedougou and Ziguinchor, which are both located in the more humid part of the country. This result is consistent with the findings of Jensen *et al.* (1990), who concluded that the Turc approach yielded best results in comparison to other radiation based methods for humid regions. In contrast to the stations located in the southern part, performance of the Turc approach was poor in the northern part, which was indicated by the corresponding  $R^2$  values of 0.07, 0.21, and 0.40 for Dakar-Yoff, Saint Louis, and Kaolack stations, respectively (Tab. 2). In conclusion it can be stated that the Turc method clearly underestimates  $ET_0$  compared to the FAO-PM model during the dry season and overestimates FAO-PM derived  $ET_0$  in the wet season. Therefore, it can be hypothesized that the original setting of the Turc formula with  $a = 0.31$ ,  $b = 2.094$ , and  $C$  constrained to be 1 at relative humidity  $\geq 50\%$  (Eq. 6, 7, and 8), are only valid under fairly humid conditions, and that for less humid conditions these values and constrain have to be adapted.

### ***Fitting Turc's parameters***

To account for the discrepancies problems of overestimated  $ET_0$  derived Turc at high humidity and underestimation  $ET_0$  with low humidity, a calibration of the Turc parameters ( $a$  and  $b$ ) was performed. The calibration was based on minimizing the sum of squared residuals (SSR) between  $ET_0$  computed by Turc approach and  $ET_0$  calculated by the FAO-PM method using EXCELS non-linear optimization routine. The results of the fitting exercise are listed in Tab. 2, whereby the general trend shows a slightly better representation of the Turc formula in comparison to the original parameters with a slightly higher  $R^2$  value and reduced RMSE. Nevertheless, the  $R^2$  did not exceed 0.63 and the overall RMSE indicates that the fitting of  $a$  and  $b$  did not solved the problem of systematic  $ET_0$  under- or overestimation, especially the systematic mismatch between the seasons. The cross plots for Turc versus FAO-PM

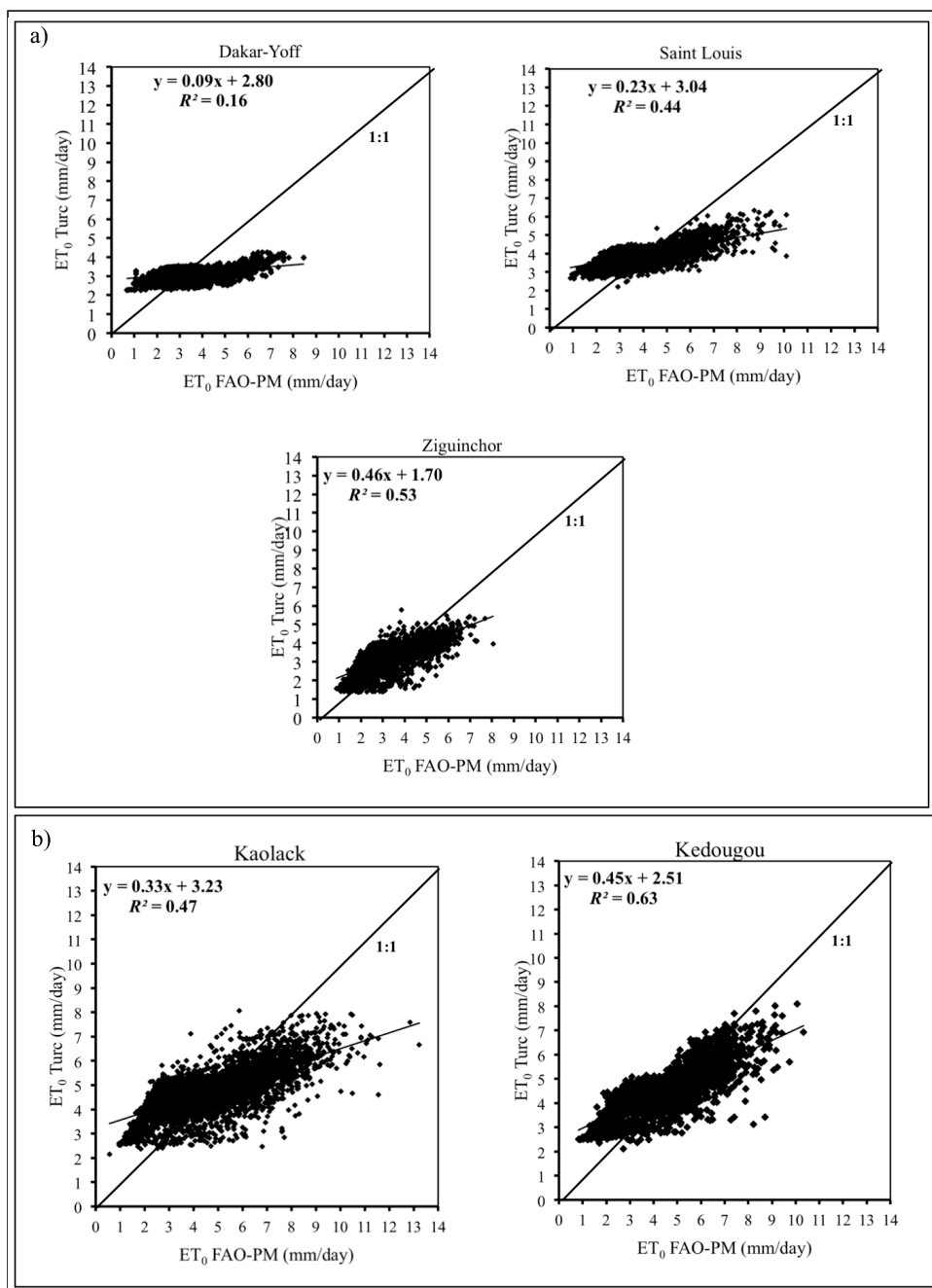


Fig. 3 - Daily  $ET_0$  estimates of FAO-PM vs. Turc (Eq. 6, 7, and 8) for the time period 2000 to 2013 after fitting a and b with constrain that  $C = 1$  for  $RH \geq 50\%$ . a) are the coastal stations and b) are the continental stations.

ET<sub>0</sub> are shown for the different stations in Fig. 3a and 3b. As can be seen the Kedougou station with highest  $R^2$  (0.63) and also the Ziguinchor station with an  $R^2$  of 0.53 scatter more or less around the 1:1 line, whereas the other stations indicate a totally different behavior. Surprisingly, the fitted  $a$  and  $b$  parameter vary strongly between the stations, whereby the  $b$  parameters varies by nearly a factor of 10 between the Ziguinchor and the Dakar-Yoff station (see Tab. 2). With respect to the original setting of  $b$  (= 2.094) all stations show more or less higher values. On the other hand, the  $a$  parameter varied only by a factor of three between the stations but are generally lower as the original value of 0.31. Nevertheless, the results listed in Tab. 2 and presented in Fig. 3 clearly show that the problem of the Turc approach cannot be solved by adjusting the two parameters  $a$  and  $b$  alone.

In a next step, not only the parameters  $a$  and  $b$  but also the constrain described for  $C$  (Eq. 6 and 7) were accounted for in the fitting. In more detail the term describing  $C$  in Eq. [7] was also parameterized introducing two additional parameters  $e$  and  $f$  and the constrain of  $C$  being equal to 1 at the threshold of  $\geq$  relative humidity was neglected. As a consequence Eq. [6] can be rewritten as:

$$ET_0 = a * \left(1 + \frac{e - RH}{f}\right) * (R_g + b) \left(\frac{T}{T + 15}\right) \quad (13)$$

Consequently four parameters ( $a$ ,  $b$ ,  $e$ , and  $f$ ) were fitted. In general, the results show a good correlation between the Turc and FAO-PM ET<sub>0</sub> values, whereby the  $R^2$  greatly improved from  $<0.63$  to  $>0.77$  for all stations (see Tab 2), whereby four out of the five stations showed an  $R^2$  exceeding 0.82. Additionally, the RMSE greatly improved by the fitting of all four parameters to 1.25, 0.84, 0.58, 0.84, and 0.54 mm day<sup>-1</sup> for Dakar-Yoff, Saint Louis, Kaolack, Ziguinchor and Kedougou stations, respectively. This means an improvement between 30 and 82 % of the values obtained with respect to the original values. The fitted values for  $a$  ranged between 0.12 for the Kedougou, 0.15 for the Dakar-Yoff and Ziguinchor, 0.17 for the Saint Louis and 0.18 for the Kaolack station, respectively. These values are slightly lower as in the original Turc formulation ( $a = 0.31$ ). However, the  $b$  values increase much larger to 21, 23, 28, 36, and 43 in Ziguinchor, Kaolack, Saint Louis, Dakar-Yoff, and Kedougou stations compared to the original setting of  $b = 2.094$ . Surprisingly, the fitted values of  $e$  and  $f$  were very close to the original values with  $e$  ranging from 47 to 56 and  $f$  from 63 and 80 compared to the original setting with  $e = 50$  and  $f = 70$ . Plotting daily FAO-PM ET<sub>0</sub> versus the fitted ET<sub>0</sub> calculated by the Turc approach with the parameters listed in Tab. 2 for each station is presented in Fig. 4. The results revealed practically perfect agreement between the FAO-PM and the Turc method. Compared to the fitting of  $a$  and  $b$  only and constraining  $C$  to be 1 at  $RH \geq 50\%$  the data are now close to the 1:1 line, which is also reflected by the large decrease in RMSE for all stations. It has to be mentioned that the adjusted  $R^2$  ( $R^2_{adj}$ ) exceeds the  $R^2$  and also the  $R^2_{adj}$  for the original formulation with given  $a$  and  $b$  parameters as well as the fitting of  $a$  and  $b$

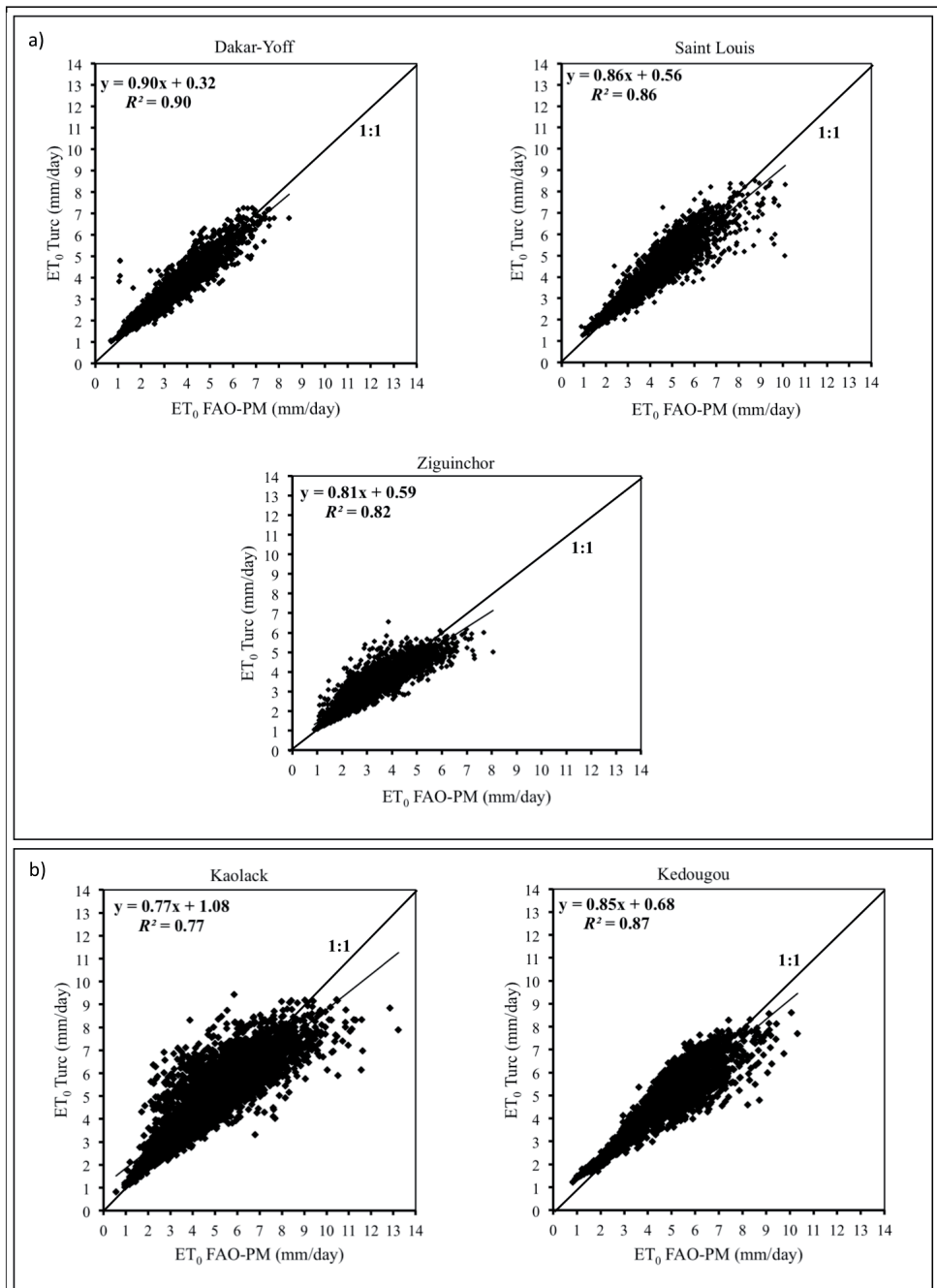


Fig. 4 - Daily  $ET_0$  estimates of FAO-PM vs. Turc (Eq. 6, 7, and 8) for the time period 2000 to 2013 after fitting a, b, e, and f according to Eq. [13]. a) are the coastal stations and b) are the continental stations.

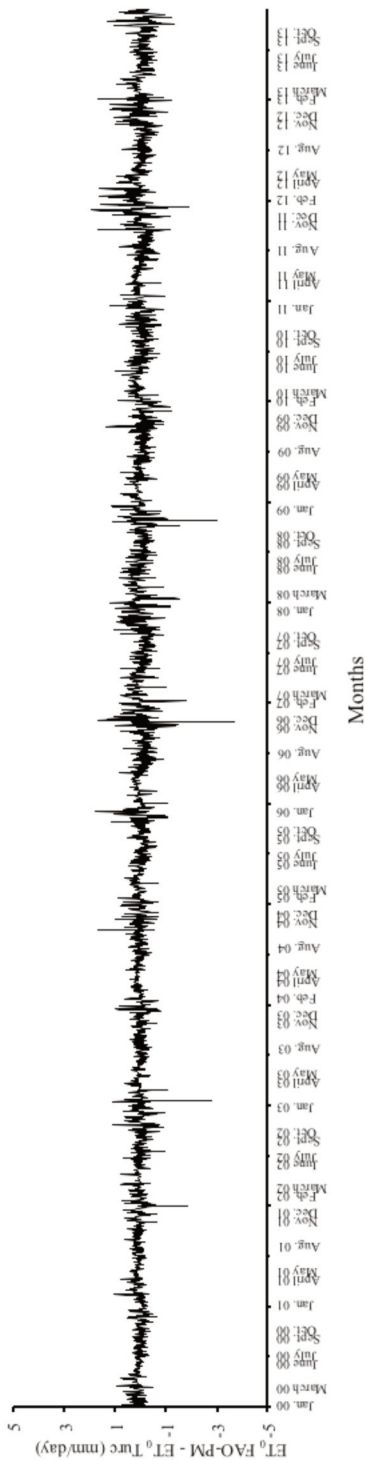


Figure 5 - Comparison between daily  $ET_0$  computed by the Turc method (Eq. 13) fitting parameters (a, b, e, and f) against those by the FAO-PM method: a) Dakar-Yoff station (atlantic influence) and b) Kaolack station (continental influence).

performed earlier. This indicates that the introducing two additional fitting parameters ( $e$  and  $f$ ) are favorable.

In Fig. 5 two examples are shown for the two stations Dakar-Yoff (oceanic influence) and Kaolack (continental), where the FAO-PM  $ET_0$  is plotted versus the Turc  $ET_0$  over the course of 14 years (2000 to 2013). As can be seen by looking at the residuals ( $ET_0$  FAO-PM minus  $ET_0$  Turc) only a small seasonal trend is detectable especially for the Kaolack station and most of the mismatch (lower panel of plot Fig. 5a and Fig. 5b) is random and were observed in the dry season.

Because fitting all four parameters indicated that parameters  $e$  and  $f$  are not far from the values originally proposed when  $C$  was not constrained by a relative humidity threshold, and therefore, the next logical step would be not to fit these two parameters but keep  $C$  to be unconstrained. This would theoretically give Eq. [13] the flexibility needed on a lower number of free parameters. The results for the fitting of the five stations without constraining  $C$  and setting  $e$  and  $f$  to the originally proposed values ( $e = 50$  and  $f = 70$ ) yielded comparable  $R^2$ ,  $R^2_{adj}$ , and RMSE as for fitting all four parameters simultaneously (see Tab. 2). Therefore, it seems logic to use the last approach and fit only the parameters  $a$  and  $b$  and do not constrain  $C$  on relative humidity ( $RH$ ).

Unfortunately, up to this point each station was fitted separately and no universal parameter set valid for all stations was deduced, which reduces the applicability for additional sparse instrumented climatic stations in Senegal. To analyze if a general parameter set exists or can be used, we fitted the two parameters  $a$  and  $b$  for one station and used these fitted parameters in a forward  $ET_0$  calculation of the remaining four stations. The station used for fitting was shifted, which means that each station was used once for fitting and prediction of all other station. Hereby,  $C$  was not constrained and  $e$  and  $f$  were set again to the originally proposed values with  $e = 50$  and  $f = 70$ . The results in term of RMSE and  $R^2$  are plotted in Fig. 6. As can be seen for example, fitting  $a$  and  $b$  for the station Dakar-Yoff yielded reasonable results for all Turc  $ET_0$  estimations for the other four stations based on the Dakar fitted parameters with  $R^2$  exceeding 0.8 and a fairly low RMSE of less than  $1.15 \text{ mm day}^{-1}$ . This also holds for all other combination shown except for the combination where the Ziguinchor station was used for fitting and the derived parameters were used for the prediction of the Kaolack station. Here, the  $R^2$  is still high for the Kaolack station with  $R^2 = 0.77$  but the RMSE increased to 1.40. To judge which station and corresponding parameters should be used for all stations shown, and any unknown station location, the resulting  $ET_0$  from Turc and FAO-PM were plotted in one graph for each reference station in Fig. 7. To indicate the different stations each station was coded with an individual color throughout all plot. Additionally,  $R^2$  and the RMSE over all stations was calculated and indicated in Fig. 7. Surprisingly,  $R^2$  calculated over all stations for all reference station combination was 0.80, and therefore, it is not

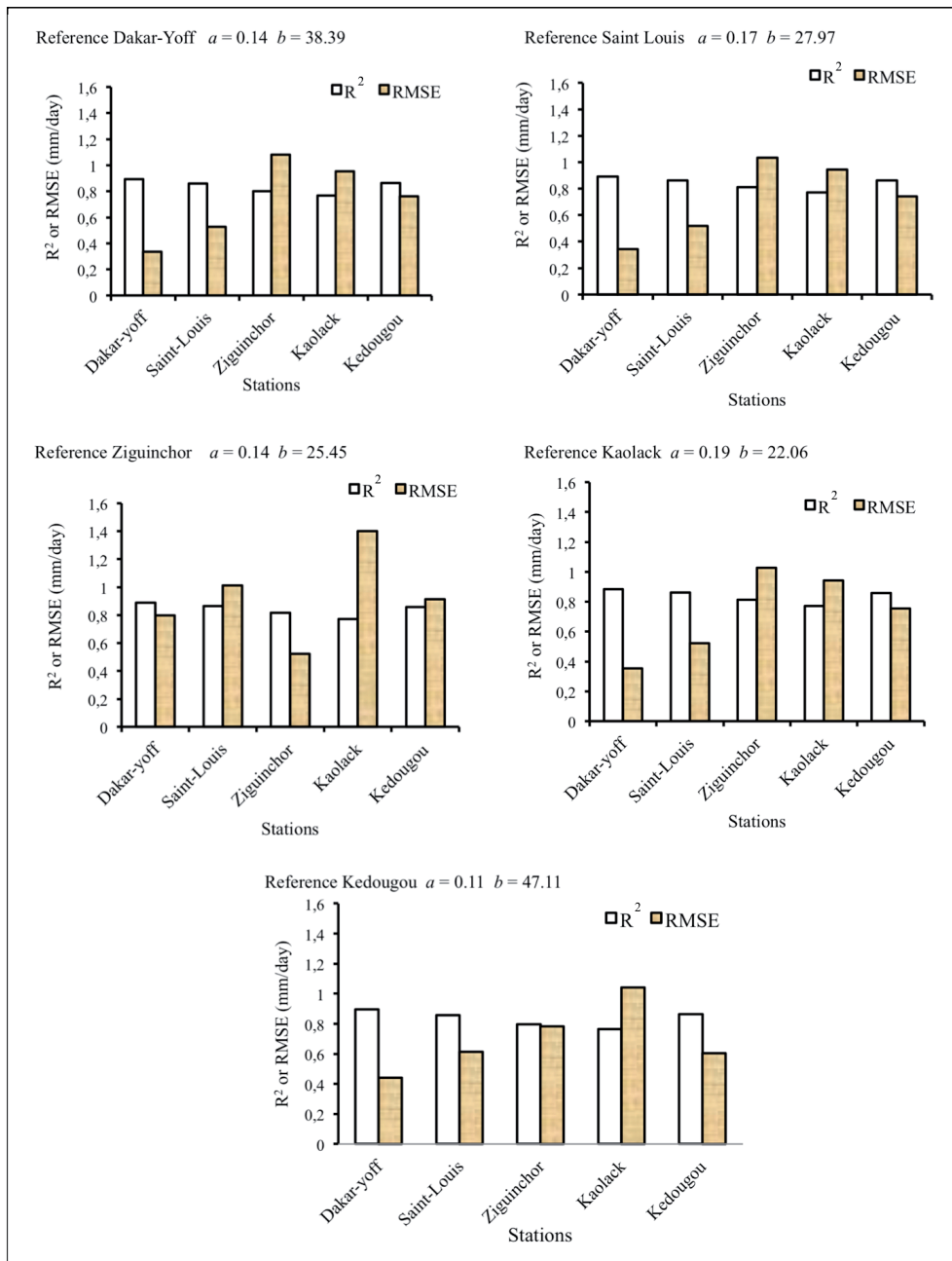


Figure 6 - Comparing  $ET_0$  computed by Turc (Eq.13) against FAO-PM method for a given station and predicting all other stations with the parameters fitted for the reference station.  $a$ ,  $b$  fitted,  $C$  not constrained, and  $e$  set to 50 and  $f$  to 70.



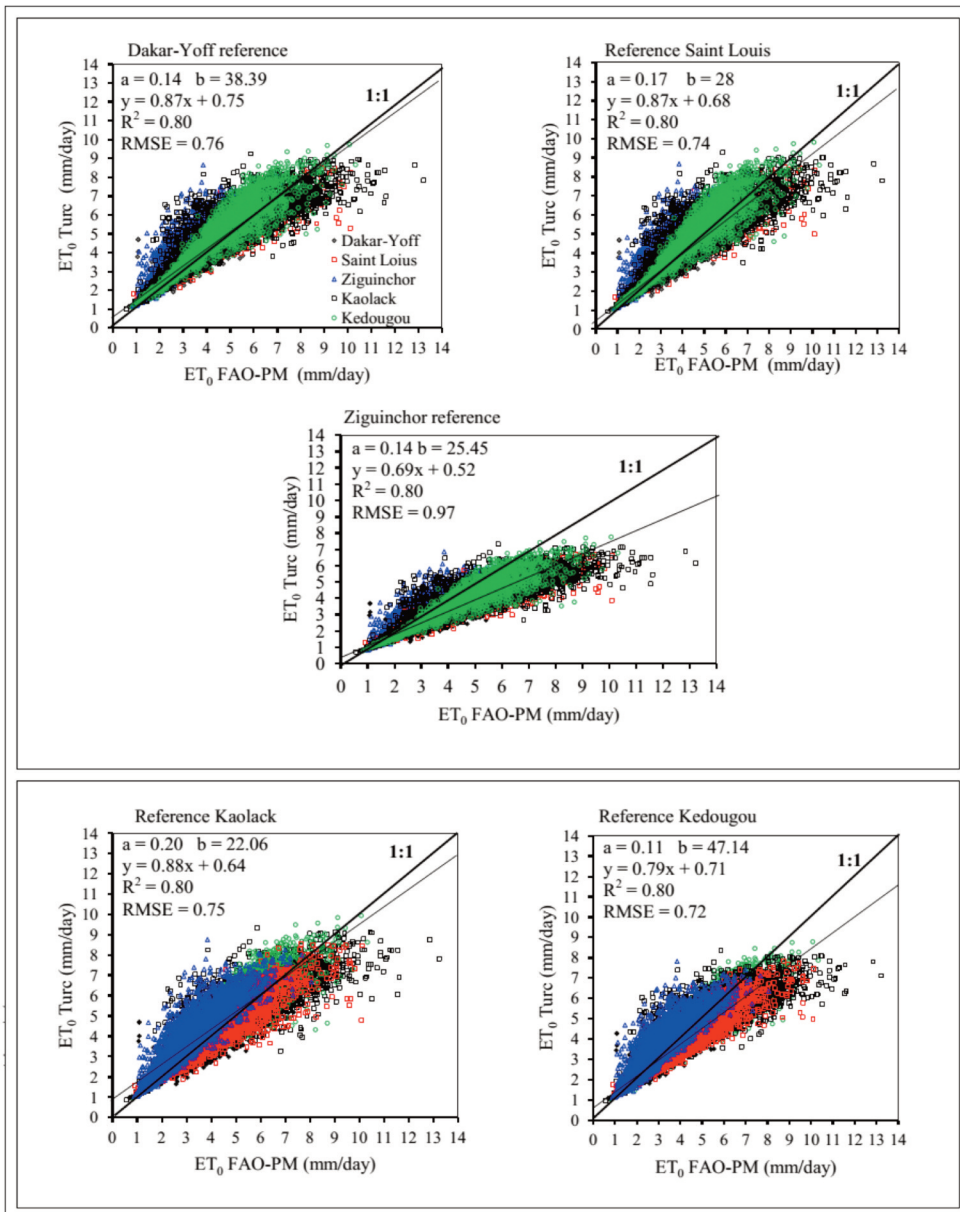


Figure 7 - Comparing  $ET_0$  computed by Turc (Eq.13) against FAO-PM method for a given station and predicting all other stations with the parameters fitted for the reference station.  $R^2$  and RMSE were calculated on all Turc  $ET_0$  versus FOA-PM  $ET_0$ .  $a$  and  $b$  fitted,  $C$  not constrained and  $e$  set to 50 and  $f$  to 70.

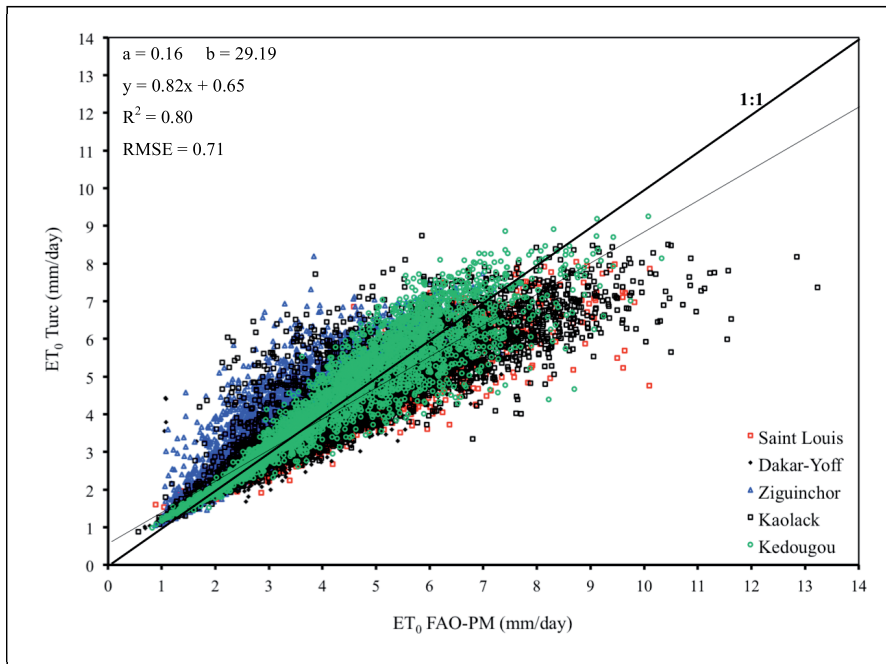


Figure 8 - Turc (Eq.13) versus FAO-PM  $ET_0$ , whereby all stations were used for fitting.  $a$  and  $b$  fitted,  $C$  not constrained and  $e$  set to 50 and  $f$  to 70.

possible to judge which reference station should be used for the entire set of stations based on the  $R^2$ .

Looking at the RMSE on the other hand, which provided information about the mean errors shown a more detailed picture, whereby the RMSE varies between 0.72 to 0.97 mm day<sup>-1</sup>. Smallest predictive error (RMSE = 0.72 mm day<sup>-1</sup>) was found for the combination using Kedougou as a reference station for fitting parameters  $a$  and  $b$  and using these parameters for all other stations and largest predictive error (RMSE = 0.97 mm day<sup>-1</sup>) was found for Ziguinchor station with fitted  $a = 0.14$  and  $b = 25.45$ . Based on the fairly small differences between the RMSE for all five reference stations depicted in Fig. 7 it seems that it is not too important whether the stations used for fitting are located close to the coast or inland.

Another possibility to get information about a unique parameter set is to fit the parameters  $a$  and  $b$  for all stations at the same time. Therefore, all station data are combined in one matrix and again the mismatch between Turc and FAO-PM  $ET_0$  was minimized. The results of this fitting approach are plotted in Fig. 8. Surprisingly, the  $R^2$  is again 0.80 but the RMSE slightly decreased to 0.71 mm day<sup>-1</sup> compared to the best combination shown in Fig. 7 with an RMSE of 0.72 mm day<sup>-1</sup>. Looking at the regression line it becomes also obvious that it slightly differs from the 1:1 line, which

means that higher potential evaporation rates are still underestimated by the Turc approach using the parameters  $a = 0.16$  and  $b = 29.19$ .

The performance of the impact of setting the parameters  $a = 0.16$ ,  $b = 29.19$  and not constraining  $C$  in each station show that the RMSE is equal to 0.37, 0.54, 0.87, 0.64, and 0.95 mm day<sup>-1</sup> in Dakar-Yoff, Saint-Louis, Ziguinchor, Kedougou, and Kaolack stations respectively.  $R^2$  exceed 0.80 for all stations except in Kaolack station where the  $R^2$  is only 0.77. Nevertheless, it seems that fitting all stations at the same time has some advantage over fitting single reference stations and using the reference station parameters for predicting all other stations. The reasons for this might be the trade-off between more coastal and more continental stations and their different reaction to the fitting parameters.

## Summary and conclusion

The Penman-Monteith equation as described in the FAO-56 (FAO-PM) (Allen *et al.*, 1998) has a strong physical background and has been proven to accurately estimate  $ET_0$  over a wide range of climatic conditions. Nevertheless, a circumstance limiting its widespread use is the high number of meteorological variables as inputs such as air temperature, wind speed, relative humidity, soil heat flux, and solar radiation. The lack of the availability of these measurements in most parts of the world has led to the development of simpler equations for the  $ET_0$  estimation, which requiring only a few and easily measureable climatic variables. One of these simple approaches is the Turc formula developed for southern France and northern Africa but already applied at different locations over the globe. Because there are strong indicators that the Turc equation does not work appropriate for all climatic conditions, we tested the applicability for a semi-arid climate with pronounced rainfall season in Senegal.

The results showed that Turc equation in its original formulation systematically underpredicts mean daily FAO-PM  $ET_0$  for the dry season and overestimated FAO-PM  $ET_0$  for the wet season. To enhance the  $ET_0$  prediction based on the Turc approach the empirical parameters  $a$  and  $b$  were fitted against FAO-PM  $ET_0$  for five selected stations located in Senegal but the prediction did not improved substantially. In a next step, the constrained term  $C$  was modified in a way to get two additional free parameters  $e$  and  $f$ , which could be also fitted in conjunction with the parameters  $a$  and  $b$ , whereby the prediction of the Turc approach increased significantly when the constrain of  $C$  being dependent on relative humidity was not accounted for. Because the fitting for the parameters  $e$  and  $f$  showed that these values are close to those originally proposed by Turc,  $e$  and  $f$  were set fixed again to the original values and only the constrain of  $C$  being dependent on relative humidity was not accounted for in further fitting. In this step, the results indicated that fitting  $a$  and  $b$  without constraining  $C$  yield comparable results as fitting all four parameters without

constraining  $C$ . This indicates that not  $a$  and  $b$  are the most critical parameters but not constraining the humidity term  $C$  greatly improves  $ET_0$  estimates in Senegal. Therefore, we propose to use the Turc approach without constraining  $C$  in the environment analyzed.

In a last step, the applicability of a single set of parameters was analyzed and it turned out that one set of parameters ( $a = 0.16$  and  $b = 29.19$ ) without constraining  $C$  on relative humidity has the ability to predict  $ET_0$  with an acceptable accuracy for all stations analyzed.

Finally, it should be noted that this study was based on a limited data set for Senegal. Therefore, further studies including long time series of climatic data from different climates and locations are desirable to prove the concepts shown western Africa or even for global application.

### Acknowledgement

This study was financially supported by a Post-Doctoral Grant from the German Academic Exchange Service (DAAD) to Ousmane Coly Diouf at Forschungszentrum Jülich GmbH, Agrosphere Institute IBG-3. The authors are grateful to the DAAD for this funding.

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